Foglio Fronte/Retro 10x15 per esame di Analisi Reale.

$X:S\to R$	R	$P(X=n)\quad f_X(t)$	$E(X)$	$Var(X)$
$B(p)$	0,1	p	p	$1-p$
Binomiale $B(n,p)$	$[0,1,\dots,n,\dots]$	$\binom{n}{k}p^k(1-p)^{n-k}$	np	$np(1-p)$
$Poiss(\lambda)$	$[0,1,\dots,n,\dots]$	$e^{-\lambda}\lambda^n/n!$	λ	λ
$N(\mu,\sigma^2)$		$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2}$	μ	σ^2
$\exp(\lambda)$		$\lambda e^{-\lambda t}\quad t\geq 0$ $0\quad t<0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
$\sigma(p)$	$[1,2,\dots,n,\dots]$	$p(1-p)^{n-1}$		

$$P(A)=P(A\cap B_1)+\cdots+P(A\cap B_n)=P(A|B_1)P(B_1)+\cdots+P(A|B_n)P(B_n)$$

$$P(A|B)=\frac{P(A\cap B)}{P(B)}\qquad P(B_k|A)=\frac{P(A|B_k)P(B_K)}{P(A|B_1)P(B_1)+\cdots+P(A|B_n)P(B_n)}$$

$$E(X)=\int_{-\infty}^{+\infty}t\,f_X(t)\,dt\qquad E(X)=\sum_{n=1}^{\infty}x_n p_n\qquad S^2=\frac{SS}{n-1}=\frac{1}{n-1}\sum_{i=1}^{n}(X_i-M_n)^2$$

$$Var(X)=E\left((X-E(X))^2\right)=E(X^2)-E^2(X)\qquad E(SS)=(n-1)\sigma^2$$

$$Var(X+Y)=Var(X)+Var(Y)+2[E(XY)-E(X)E(Y)]\qquad M_n=\frac{X_1+\cdots+X_n}{n}$$

$$\lim_{n\to\infty}P\{a\leq X_n\leq b\}=\int_a^b\frac{1}{\sqrt{2\pi}}e^{-t^2/2}dt=\Phi(b)-\Phi(a)\qquad E(M_n)=\mu$$

$$S_{XY}=\frac{1}{n-1}\sum_{i=1}^{n}(X_i-M_X)(X_i-M_Y)\qquad \rho_{XY}=\frac{COV(X,Y)}{\sigma_x\sigma_Y}\qquad Var(M_n)=\frac{\sigma^2}{n}\qquad a=M_X-bM_Y$$

$$COV(X,Y)=E[(X-\mu_X)(Y-\mu_Y)]=E(XY)-E(X)E(Y)\qquad Z=\frac{X-\mu}{\sigma}\qquad b=\frac{S_{XY}}{S_X^2}$$

Q.Pivotale (QP)	Metodo	I_α
$S_n=\frac{\sqrt{n}}{\sigma}(M_n-\mu)$ Lim.Centrale	La QP tende in prob alla Normale STD	$P\{-z_{\alpha/2}<S_n<z_{\alpha/2}\}=1-\alpha$ $I_\alpha=\left(M_n-\frac{\sigma}{\sqrt{n}}z_{\alpha/2};M_n+\frac{\sigma}{\sqrt{n}}z_{\alpha/2}\right)$
$S_n=\frac{\sqrt{n}}{S}(M_n-\mu)$	La QP è la T_n-1 di Student	$P\{-q_{\alpha/2}<S_n<q_{\alpha/2}\}=1-\alpha$ $I_\alpha=\left(M_n-\frac{S}{\sqrt{n}}q_{\alpha/2};M_n+\frac{S}{\sqrt{n}}q_{\alpha/2}\right)$
$C_{n-1}=\frac{n-1}{\sigma^2}S^2$ Teo. Fisher	La QP si comporta per $n\to\infty$ come la χ^2_{n-1}	$P\{q_{\alpha/2}<C_{n-1}<q_{1-\alpha/2}\}=1-\alpha$ $I_\alpha=\left(\frac{(n-1)s^2}{q_{1-\alpha/2}};\frac{(n-1)s^2}{q_{\alpha/2}}\right)$

Per Paerson sappiamo che la D_{N-1} tende alla χ^2_{N-1} per n grande.

N Num. valori assunti dalla Y $\quad$ n Num. campioni della X

n_i Freq. della Y in base alla sua legge e riferite a n $\Rightarrow$ $n_i=np_i$

ν_i Freq. che il campione assume val. y_i $\Rightarrow$ $\nu_i=Card\{x_j|x_j=y_i\}$

$$D_{N-1}=\sum_{i=1}^{N}\frac{n}{p_i}\left(\frac{\nu_i}{n}-p_i\right)^2=\sum_{i=1}^{N}\frac{(\nu_i-n_i)^2}{n_i}\qquad I_\alpha=(0,q_{1-\alpha})$$

Cebicev $P\{|X-\mu|\geq\eta\}\leq\frac{\sigma^2}{\eta^2}\qquad \int_{-\infty}^{\infty}(t-\mu)^2f_X(t)dt$

Numeri grandi $P\{|M_n-\mu|\geq\eta\}\leq\frac{\sigma^2}{n\eta^2}$

$$E(SS)=E\left[\sum_{i=1}^{n}(X_i-\mu+\mu-M_n)^2\right]$$

$$\sum_i(x_i+y_i)P_\xi(x_i,y_i)=\sum_j\sum_k(x_j+y_k)P_\xi(x_j,y_k)=\sum_j\left(\sum_k x_jP_\xi(x_j,y_k)\right)+\sum_k\left(\sum_j y_kP_\xi(x_j,y_k)\right)=\sum_j x_jP^X_{x_j}+\sum_k y_kP^Y_{y_k}$$

$$P\{X\in I_1,Y\in I_2\}=P\{X\in I_1\}P\{Y\in I_2\}\Rightarrow\int_{I_1}\int_{I_2}f_\xi(t,s)\,dtds=\int_{I_1}f_X(t)dt\int_{I_2}f_Y(s)ds$$

$$\hat\theta\to\theta\ \ in\ \ prob.\Leftrightarrow\lim_{n\to\infty}P\{|\hat\theta-\theta|>\eta\}=0$$

$$A=\{s\in S\ \ t.c.\ \ |\hat\theta-\theta|\leq\eta\}\qquad B=\{s\in S\ \ t.c.\ \ |\hat\theta-E(\hat\theta)|\leq\eta/2\}$$

$$se\ \ n>n_0\Rightarrow B\subset A\qquad \underbrace{|\hat\theta-\theta|}_{\eta}\leq\underbrace{|\hat\theta-E(\hat\theta)|}_{\eta/2}+\underbrace{|E(\hat\theta)-\theta|}_{\eta/2}$$

$$Cebicev\Rightarrow P\{|\hat\theta-E(\hat\theta)|>\eta/2\}\leq\frac{Var(\hat\theta)}{(\eta/2)^2}\to 0\ \ se\ \ n\to\infty$$

$$\forall\epsilon>0\ \exists n_1\ \ t.c.\ \ se\ \ n\geq n_1\Rightarrow P\{s\in S\ \ t.c.\ \ |\hat\theta-E(\hat\theta)|>\eta/2\}<\epsilon$$

$$P\underbrace{\{s\in S\ \ t.c.\ \ |\hat\theta-E(\hat\theta)|\leq\eta/2\}}_{B}\geq 1-\epsilon$$

$$se\ \ n>max(n_0,n_1)\ \Rightarrow\ P(B)\geq 1-\epsilon\ \Rightarrow\ P(A)\geq 1-\epsilon$$

$$\Rightarrow\ \forall\epsilon>0\ \exists n_2=max(n_0,n_1)\ \ t.c.\ \ se\ \ n\geq n_2\Rightarrow P(A)\geq 1-\epsilon$$

$$\lim_{n\to\infty}P\{|\hat\theta-\theta|\leq\eta\}=1\quad\Rightarrow\quad\lim_{n\to\infty}P\{|\hat\theta-\theta|>\eta\}=0$$

$$f_{T_n}(t)=\frac{\Gamma((n+1)/2)}{\Gamma(n/2)\sqrt{n\pi}}\left(1+\frac{t^2}{n}\right)^{-(n+1)/2}\qquad f_{\chi^2_n}=\begin{cases}\frac{(1/2)^n}{\Gamma(n/2)}e^{-t^2/2}t^{n/2-1} & t>0\\ 0 & t\leq 0\end{cases}$$

$$r_{XY}=\frac{\frac{1}{n-1}\sum_{i=1}^{n}(X_i-M_X)(Y_i-M_Y)}{S_XS_Y}\qquad Z=\frac{\sqrt{n-3}}{2}\ln\left[\frac{(1+r_{XY})(1-\rho_{XY})}{(1-r_{XY})(1+\rho_{XY})}\right]$$